1 (a) Naturally occurring uranium consists of two isotopes in the ratio ${}^{235}_{92}\mathrm{U}/{}^{238}_{92}\mathrm{U} = 0.73\%$. If the two isotopes existed in equal amounts at the time the earth was formed, calculate the age of the earth. $\{\mathbf{5}\}$

[The mean lifetime of ${}^{235}_{92}\mathrm{U}$ is 1.02×10^9 years and ${}^{238}_{92}\mathrm{U}$ is 6.45×10^9 years]

(b) Converters are a type of nuclear reactor that can produce fissionable fuel. In a converter, ${}^{239}_{92}\mathrm{U}$ decays via β^- decay to ${}^{239}_{93}\mathrm{Np}$, with a half-life of 23 minutes. ${}^{239}_{93}\mathrm{Np}$ then decays via β^- decay to ${}^{239}_{94}\mathrm{Pu}$, with a half-life of 2.3 days. A sample of pure ${}^{239}_{92}\mathrm{U}$ is produced at $t = 0$. What is the ratio of ${}^{239}_{93}\mathrm{Np}/{}^{239}_{92}\mathrm{U}$ after 6 hours? $\{\mathbf{7}\}$

[Hint: write down decay rate equations for ${}^{239}_{92}\mathrm{U}$ and ${}^{239}_{93}\mathrm{Np}$ and solve these e.g. using an integrating factor. You can assume ${}^{239}_{94}\mathrm{Pu}$ is stable and does not decay]

(c) A nuclide with $Z = 18$ and $A = 37$ is radioactive. From the data given below, determine whether it decays by β^- emission, β^+ emission or orbital electron capture, by calculating the energy release Q in each case. $\{\mathbf{7}\}$

[The binding energy of ${}^{37}_{18}\mathrm{Ar}$ is $315.5087\,\mathrm{MeV}$, ${}^{37}_{17}\mathrm{Cl}$ is $317.1042\,\mathrm{MeV}$ and ${}^{37}_{19}\mathrm{K}$ is $308.5763\,\mathrm{MeV}$. The mass of the proton is $938.2721\,\mathrm{MeV/c^2}$, the mass of the neutron is $939.5654\,\mathrm{MeV/c^2}$ and the mass of the electron is $0.5110\,\mathrm{MeV/c^2}$]

(d) In the shell model, the spin of a nucleus with a single valance nucleon is determined by the total angular momentum j of this nucleon. With the aid of diagrams, show the filling of shells for protons and neutrons and hence determine the spin for ${}^{41}_{20}\mathrm{Ca}$. $\{\mathbf{3}\}$

[The shell model is shown in Fig. Q1 over the page]

(e) In D-D fusion, the deuterons need to come within $100\,\mathrm{fm}$ of each other to overcome the Coulomb barrier. If the energy is supplied in the form of thermal energy of the deuterons, what temperature must the plasma be at? Give your answer in Kelvin. $\{\mathbf{3}\}$

[The Boltzmann constant is $k = 1.381 \times 10^{-23}\,\mathrm{J\,K^{-1}}$]

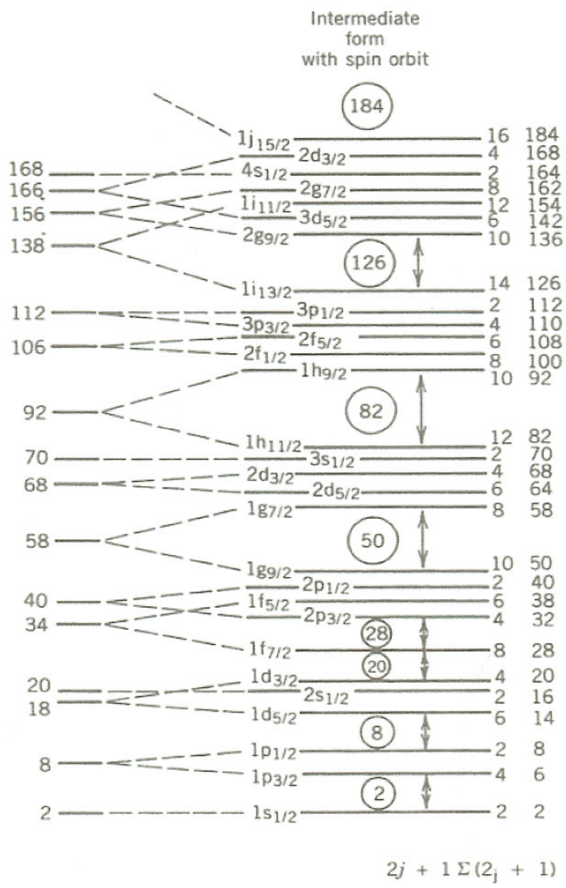

Fig. Q1: Nuclear Shell Model